

Question 1

For

$$\mathbf{F}(x, y, z) = \langle x^2y, yz^2, xz \rangle,$$

compute $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.

Tasks

1. Compute the divergence term by term.
2. Compute all three components of the curl.
3. Evaluate both quantities at $(1, 2, -1)$.

Question 1 – Solution

Strategy. Label the components P, Q, R and apply the divergence and curl formulas in a fixed order.

Step 1: Divergence With $P = x^2y$, $Q = yz^2$, and $R = xz$,

$$\nabla \cdot \mathbf{F} = P_x + Q_y + R_z = 2xy + z^2 + x.$$

Thus

$$\boxed{\nabla \cdot \mathbf{F} = 2xy + z^2 + x}.$$

Step 2: Curl

$$\begin{aligned}\nabla \times \mathbf{F} &= \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle \\ &= \langle 0 - 2yz, 0 - z, 0 - x^2 \rangle.\end{aligned}$$

Therefore

$$\boxed{\nabla \times \mathbf{F} = \langle -2yz, -z, -x^2 \rangle}.$$

Step 3: Evaluate At $(1, 2, -1)$,

$$\boxed{\nabla \cdot \mathbf{F} = 6}, \quad \boxed{\nabla \times \mathbf{F} = \langle 4, 1, -1 \rangle}.$$

Verification The curl components follow the cyclic order $(R_y - Q_z, P_z - R_x, Q_x - P_y)$; recomputing from the determinant form gives the same signs.