

Question 10

Let D be the ellipse bounded counterclockwise by

$$x = 2 \cos t, \quad y = 3 \sin t, \quad 0 \leq t \leq 2\pi.$$

Evaluate

$$\iint_D (3x^2 - 2y) \, dA$$

by converting it to a boundary integral.

Tasks

1. Choose P, Q so that $Q_x - P_y = 3x^2 - 2y$.
2. Parametrize the resulting boundary integral.
3. Evaluate using symmetry and a standard trigonometric integral.

Question 10 – Solution

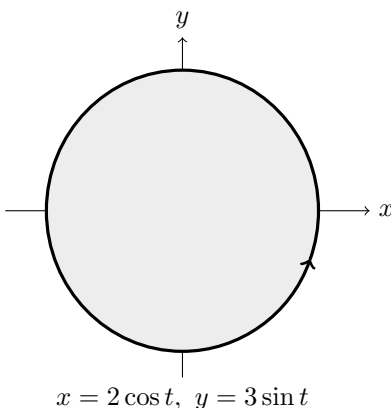
Strategy. Reverse Green's Theorem with $P = y^2$ and $Q = x^3$, then use the supplied ellipse parametrization.

Step 1: Choose a boundary field Set

$$P = y^2, \quad Q = x^3.$$

Then $Q_x - P_y = 3x^2 - 2y$, so

$$\iint_D (3x^2 - 2y) dA = \oint_C y^2 dx + x^3 dy.$$



Step 2: Substitute the boundary Since $dx = -2 \sin t dt$ and $dy = 3 \cos t dt$,

$$\oint_C y^2 dx + x^3 dy = \int_0^{2\pi} [-18 \sin^3 t + 24 \cos^4 t] dt.$$

The sine-cubed term integrates to zero over a full period, while

$$\int_0^{2\pi} \cos^4 t dt = \frac{3\pi}{4}.$$

Therefore

$$\boxed{\iint_D (3x^2 - 2y) dA = 24 \left(\frac{3\pi}{4} \right) = 18\pi}.$$

Verification The $-2y$ term is odd across the x -axis and contributes zero. Also $3x^2 \geq 0$, so the positive result has the required sign.