

Question 7

Let C be the counterclockwise triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 1)$. For

$$P(x, y) = -y^2, \quad Q(x, y) = x^2,$$

verify Green's Theorem by computing $\oint_C P dx + Q dy$ both directly and as a double integral.

Tasks

1. Evaluate all three boundary segments directly.
2. Evaluate $\iint_D (Q_x - P_y) dA$.
3. Compare the two results.

Question 7 – Solution

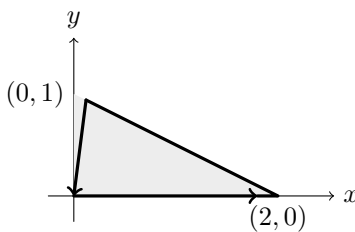
Strategy. Only the slanted edge contributes directly; then use $0 \leq y \leq 1 - x/2$ for the interior.

Step 1: Boundary calculation The horizontal and vertical coordinate-axis edges contribute zero. Parametrize the slanted edge from $(2, 0)$ to $(0, 1)$ by

$$\mathbf{r}(t) = \langle 2 - 2t, t \rangle, \quad 0 \leq t \leq 1.$$

Then $dx = -2dt$, $dy = dt$, and

$$\int_{\text{slant}} P dx + Q dy = \int_0^1 [2t^2 + 4(1 - t)^2] dt = \boxed{2}.$$



Step 2: Interior calculation Since $Q_x - P_y = 2x + 2y$,

$$\begin{aligned} \iint_D (2x + 2y) dA &= \int_0^2 \int_0^{1-x/2} (2x + 2y) dy dx \\ &= \int_0^2 \left(1 + x - \frac{3x^2}{4} \right) dx = \boxed{2}. \end{aligned}$$

Verification Both methods give 2. The slanted parametrization runs from $(2, 0)$ to $(0, 1)$, which is the required counterclockwise direction after traversing the bottom edge.