

Question 3

The planar field

$$\mathbf{F}(x, y) = \langle x^2, xy \rangle$$

acts on the disk $D : (x - 1)^2 + y^2 \leq 1$. Let $C = \partial D$ be counterclockwise. Find the outward flux of $\mathbf{F}$ across C .

Tasks

1. State the flux form of Green's Theorem.
2. Compute the divergence and its integral over D .
3. Use symmetry or the centroid of the disk to finish.

Question 3 – Solution

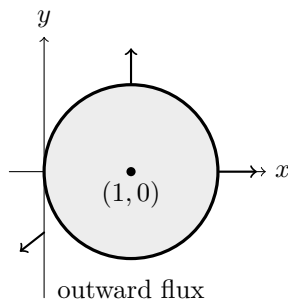
Strategy. Convert outward flux to the double integral of the planar divergence.

Step 1: Flux form For $\mathbf{F} = \langle M, N \rangle$,

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C M \, dy - N \, dx = \iint_D (M_x + N_y) \, dA.$$

Here

$$M_x + N_y = 2x + x = 3x.$$



Step 2: Integrate The disk has area π and centroid $(1, 0)$. Therefore its average x -coordinate is 1, so

$$\iint_D x \, dA = 1 \cdot \pi = \pi.$$

Consequently,

$$\boxed{\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_D 3x \, dA = 3\pi.}$$

Verification The disk lies in $0 \leq x \leq 2$, so the divergence $3x$ is nonnegative. A positive outward flux is therefore consistent with the sign of the integrand.