

Question 2

Let C be the counterclockwise ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

Evaluate

$$\frac{1}{2} \oint_C (-y \, dx + x \, dy)$$

and explain its geometric meaning.

Tasks

1. Apply Green's Theorem to the given integral.
2. Compute the enclosed area.
3. Explain the effect of reversing the ellipse's orientation.

Question 2 – Solution

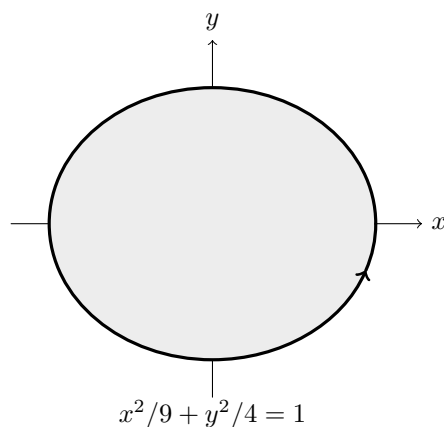
Strategy. The chosen coefficients make $Q_x - P_y = 1$, so the line integral measures area.

Step 1: Set P and Q Write

$$P = -\frac{y}{2}, \quad Q = \frac{x}{2}.$$

Then

$$Q_x - P_y = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1.$$



Step 2: Apply Green's Theorem

$$\frac{1}{2} \oint_C (-y \, dx + x \, dy) = \iint_D 1 \, dA = \text{Area}(D).$$

The ellipse has semiaxes 3 and 2, so

$$\boxed{\frac{1}{2} \oint_C (-y \, dx + x \, dy) = \pi(3)(2) = 6\pi}.$$

Step 3: Reverse orientation Clockwise traversal negates the line integral, producing -6π . Geometric area itself remains 6π ; the sign records orientation.

Verification The area formula reduces to πab , and substituting $a = 3$, $b = 2$ confirms 6π .