

Question 1

Let C be the positively oriented boundary of the rectangle

$$D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 3\}.$$

Evaluate

$$\oint_C (x^2 - y) dx + (x + y^2) dy$$

using Green's Theorem.

Tasks

1. Identify P , Q , and the required derivative difference.
2. Convert the line integral to a double integral.
3. Evaluate and check the positive orientation.

Question 1 – Solution

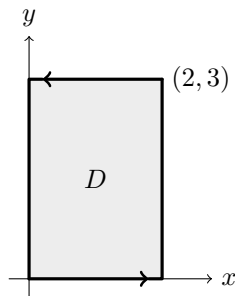
Strategy. Apply the circulation form $\oint_C P dx + Q dy = \iint_D (Q_x - P_y) dA$.

Step 1: Differentiate Here

$$P = x^2 - y, \quad Q = x + y^2,$$

so

$$Q_x - P_y = 1 - (-1) = 2.$$



Step 2: Integrate over the region

$$\oint_C P dx + Q dy = \iint_D 2 dA = 2 \text{Area}(D) = 2(2)(3) = \boxed{12}.$$

Step 3: Orientation Positive orientation means counterclockwise traversal, with the region on the left. That is the orientation for which Green's Theorem gives the displayed positive sign.

Verification The derivative difference is the constant 2 and the rectangle has area 6, so the value must be 12; reversing C would give -12 .