

Question 5

Restrict the field

$$\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

to the right half-plane $H = \{(x, y) : x > 0\}$.

Tasks

1. Find a potential function on H .
2. Verify its gradient.
3. Evaluate the line integral from $A = (1, 1)$ to $B = (\sqrt{3}, 1)$ along any path in H .

Question 5 – Solution

Strategy. The right half-plane permits a single-valued angle function with no branch crossing.

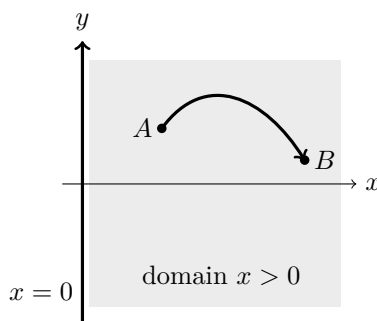
Step 1: Choose the angle potential On $x > 0$, define

$$f(x, y) = \arctan\left(\frac{y}{x}\right).$$

Using the chain rule,

$$f_x = \frac{-y}{x^2 + y^2}, \quad f_y = \frac{x}{x^2 + y^2}.$$

Hence $\nabla f = \mathbf{F}$ on H .



Step 2: Evaluate by endpoints The endpoint angles in the branch $(-\pi/2, \pi/2)$ are

$$f(A) = \arctan(1) = \frac{\pi}{4}, \quad f(B) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}.$$

Therefore

$$\boxed{\int_C \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A) = -\frac{\pi}{12}}.$$

Verification Both endpoints and the entire allowed path lie in $x > 0$, where the selected arctangent branch is continuous. The negative sign matches the decrease in polar angle from $\pi/4$ to $\pi/6$.