

Question 1

Determine whether

$$\mathbf{F}(x, y) = \langle 2xy + 3, x^2 + 4y \rangle$$

is conservative on $\mathbb{R}^2$. If it is, find a potential function f .

Tasks

1. Compare the relevant mixed partial derivatives.
2. Construct f and determine the one-variable correction term.
3. Differentiate the result to verify the field.

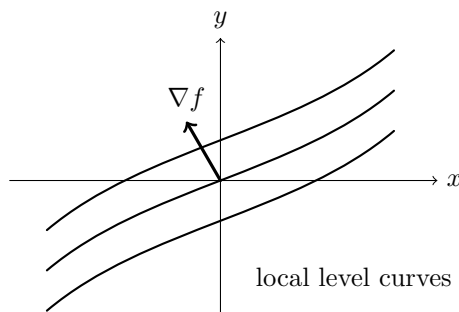
Question 1 – Solution

Strategy. Test $P_y = Q_x$ on the simply connected domain, then integrate one component and recover the missing function.

Step 1: Test the field With $P = 2xy + 3$ and $Q = x^2 + 4y$,

$$P_y = 2x, \quad Q_x = 2x.$$

The components have continuous first partial derivatives on all of $\mathbb{R}^2$, which is simply connected. Thus the equality guarantees that $\mathbf{F}$ is conservative.



Step 2: Construct a potential Integrating $f_x = P$ with respect to x gives

$$f = x^2y + 3x + g(y).$$

Then $f_y = x^2 + g'(y) = Q = x^2 + 4y$, so $g'(y) = 4y$ and $g(y) = 2y^2 + C$.

$$\boxed{f(x, y) = x^2y + 3x + 2y^2 + C}.$$

Verification Differentiation gives $\nabla f = \langle 2xy + 3, x^2 + 4y \rangle = \mathbf{F}$. The arbitrary constant has zero gradient.