

Question 9

A force is derived from the potential energy

$$U(x, y) = x^2 + 2y^2$$

by $\mathbf{F} = -\nabla U$. A particle moves along any smooth curve from $A = (2, 1)$ to $B = (0, 3)$.

Tasks

1. Compute the work $W = \int_C \mathbf{F} \cdot d\mathbf{r}$.
2. Relate the work to the change in potential energy.
3. Explain the sign of the answer.

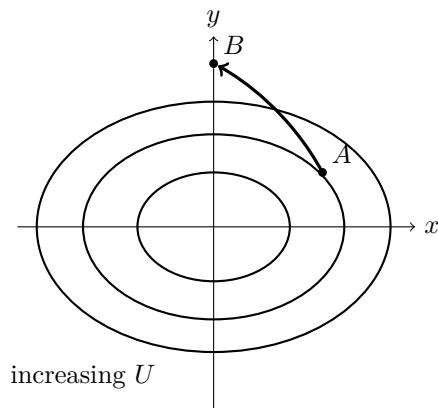
Question 9 – Solution

Strategy. Apply the theorem to $-U$: work by $-\nabla U$ is the negative of the change in U .

Step 1: Potential-energy change

$$U(A) = 2^2 + 2(1^2) = 6, \quad U(B) = 0^2 + 2(3^2) = 18.$$

Therefore $\Delta U = U(B) - U(A) = 12$.



Step 2: Work Since $\mathbf{F} = \nabla(-U)$,

$$\begin{aligned} W &= (-U)(B) - (-U)(A) \\ &= U(A) - U(B) = 6 - 18 = \boxed{-12}. \end{aligned}$$

Equivalently, $W = -\Delta U = -12$.

Step 3: Interpret the sign The particle ends at a point with greater potential energy. The force derived from $-\nabla U$ therefore does negative net work during the motion.

Verification The identity $W + \Delta U = -12 + 12 = 0$ checks the sign. Reversing the trip would produce work $+12$.