

Question 6

Let $f(x, y) = x^2 + y^2$. A curve begins at $A = (1, 2)$ and ends at a first-quadrant point B on the line $y = x + 1$. Suppose

$$\int_C \nabla f \cdot d\mathbf{r} = 20.$$

Determine B .

Tasks

1. Convert the work condition into a level-curve condition on B .
2. Solve the line-circle intersection equation.
3. Use the quadrant restriction to select and verify the endpoint.

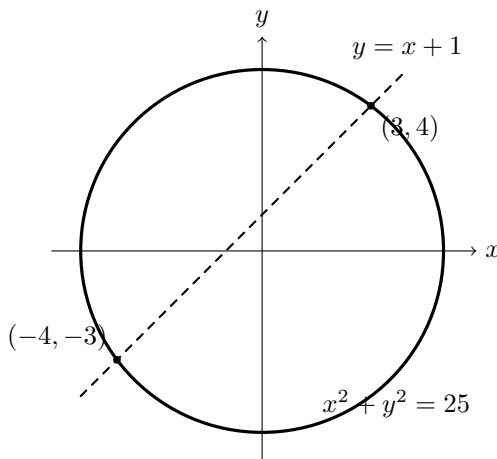
Question 6 – Solution

Strategy. Use the theorem in reverse: the known integral determines the terminal potential value.

Step 1: Determine the level Since $f(A) = 1^2 + 2^2 = 5$,

$$20 = f(B) - 5, \quad f(B) = 25.$$

Thus B lies on $x^2 + y^2 = 25$ as well as $y = x + 1$.



Step 2: Solve the intersection Substitute $y = x + 1$:

$$x^2 + (x + 1)^2 = 25 \implies 2x^2 + 2x - 24 = 0 \implies (x - 3)(x + 4) = 0.$$

The candidates are $(3, 4)$ and $(-4, -3)$.

Step 3: Apply the restriction Only $(3, 4)$ lies in the first quadrant, so

$$\boxed{B = (3, 4)}.$$

Verification The line condition gives $4 = 3 + 1$, and $f(B) - f(A) = 25 - 5 = 20$, exactly the specified integral.