

**Question 4**

Let  $f(x, y) = xe^y$ . A curve  $C$  is oriented from  $A = (0, 0)$  to  $B = (2, \ln 3)$ , and  $-C$  denotes the same curve with the reverse orientation.

**Tasks**

1. Compute  $\int_C \nabla f \cdot d\mathbf{r}$ .
2. Compute  $\int_{-C} \nabla f \cdot d\mathbf{r}$ .
3. Explain the sign change using both endpoints and orientation reversal.

**Question 4 – Solution**

**Strategy.** Evaluate the same endpoint difference in the two opposite orders.

**Step 1: Forward orientation**

$$f(A) = 0e^0 = 0, \quad f(B) = 2e^{\ln 3} = 6.$$

Thus

$$\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A) = \boxed{6}.$$

**Step 2: Reverse orientation** The initial point of  $-C$  is  $B$  and its terminal point is  $A$ , so

$$\int_{-C} \nabla f \cdot d\mathbf{r} = f(A) - f(B) = \boxed{-6}.$$

**Step 3: Connect the rules** Swapping the endpoints reverses the subtraction. This is exactly the general orientation rule

$$\int_{-C} \mathbf{F} \cdot d\mathbf{r} = - \int_C \mathbf{F} \cdot d\mathbf{r}.$$

**Verification** Adding the forward and reverse traversals makes a closed trip, and  $6 + (-6) = 0$ , as the theorem requires for a gradient field.