

Question 2

Let $f(x, y) = x^2 + y^2$. A smooth curve C travels from $A = (3, 4)$ to $B = (0, 5)$.

Tasks

1. Evaluate $\int_C \nabla f \cdot d\mathbf{r}$.
2. Interpret the result using level curves of f .
3. Decide whether a different smooth path from A to B could change the value.

Question 2 – Solution

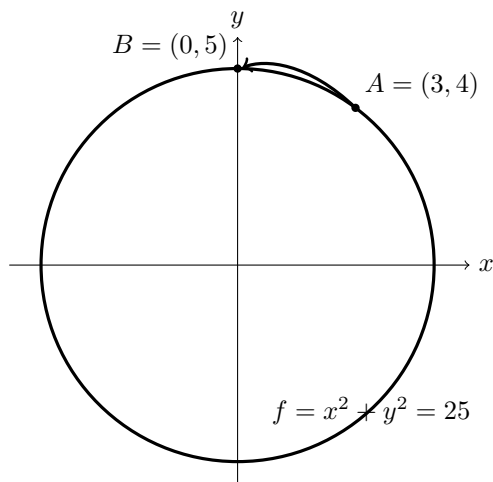
Strategy. Compare the potential values at the endpoints before doing any curve calculation.

Step 1: Endpoint values

$$f(A) = 3^2 + 4^2 = 25, \quad f(B) = 0^2 + 5^2 = 25.$$

Hence

$$\boxed{\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A) = 0.}$$



Step 2: Interpret Both endpoints lie on the level circle $x^2 + y^2 = 25$. The curve between them need not stay on that circle: f may rise and fall along the way, but its total change is still zero.

Step 3: Compare paths Any other smooth path with the same orientation from A to B gives the same endpoint difference, so it cannot change the integral.

Verification The result is not claiming that ∇f vanishes. Indeed $\nabla f = \langle 2x, 2y \rangle$ is nonzero at both endpoints; only the net change in f is zero.