

Question 1

Let

$$f(x, y, z) = x^2y + e^z,$$

and let C be any smooth curve from $A = (1, 0, 0)$ to $B = (2, 1, \ln 2)$. Evaluate

$$\int_C \nabla f \cdot d\mathbf{r}.$$

Tasks

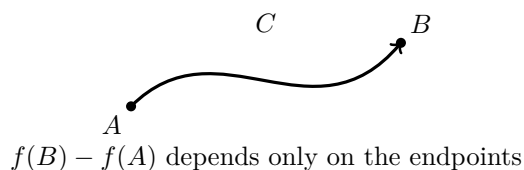
1. State the Fundamental Theorem for Line Integrals.
2. Evaluate the integral without parametrizing C .
3. Explain why the unspecified shape of C is irrelevant.

Question 1 – Solution

Strategy. Since the field is explicitly ∇f , apply the endpoint formula $\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A)$.

Step 1: Evaluate the endpoint values

$$f(B) = 2^2(1) + e^{\ln 2} = 4 + 2 = 6, \quad f(A) = 1^2(0) + e^0 = 1.$$



Step 2: Apply the theorem

$$\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A) = 6 - 1 = \boxed{5}.$$

Step 3: Explain path independence Along any smooth parametrization $\mathbf{r}(t)$, $\nabla f(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$ is the derivative of $f(\mathbf{r}(t))$. Its integral therefore records only the net change in f .

Verification The logarithmic coordinate was chosen so that $e^{\ln 2} = 2$; using $\ln 2$ itself in $f(B)$ would be an endpoint-substitution error.