

Question 10

On the counterclockwise circle $C : x^2 + y^2 = 9$, consider

$$\mathbf{F}(x, y) = \langle x - 2y, 2x + y \rangle.$$

Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by separating $\mathbf{F}$ into radial and tangential parts.

Tasks

1. Decompose the field into radial and tangential vectors.
2. Determine the work contributed by each part.
3. Verify the answer by direct parametrization.

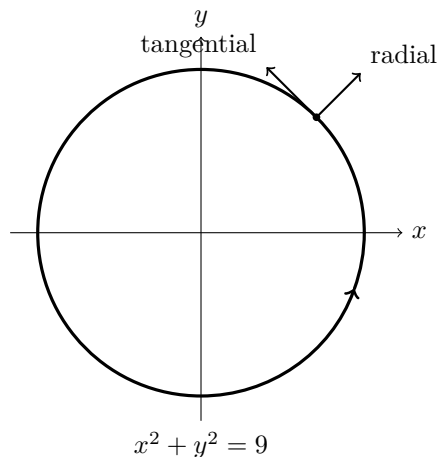
Question 10 – Solution

Strategy. Write the field as $\langle x, y \rangle + 2\langle -y, x \rangle$; on a circle the first part is normal and the second is tangent.

Step 1: Decompose

$$\mathbf{F} = \underbrace{\langle x, y \rangle}_{\text{radial}} + \underbrace{2\langle -y, x \rangle}_{\text{counterclockwise tangential}}.$$

The radial part is perpendicular to the circle's tangent, so it contributes zero work.



On the radius-3 circle, $\langle -y, x \rangle$ has magnitude 3 in the positive tangent direction. Hence the doubled part has tangential magnitude 6. Multiplying by the circumference 6π gives

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = 0 + 6(6\pi) = \boxed{36\pi}.$$

Step 2: Direct verification Set $\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t \rangle$, $0 \leq t \leq 2\pi$. Then

$$\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = 18,$$

because the radial terms cancel and the tangential terms sum to $18(\sin^2 t + \cos^2 t)$. Therefore

$$\int_0^{2\pi} 18 \, dt = 36\pi,$$

confirming both the magnitude and the positive orientation.