

Question 8

Let C be the counterclockwise boundary of the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$. For

$$\mathbf{F}(x, y) = \langle y^2, x \rangle,$$

compute the circulation $\oint_C \mathbf{F} \cdot d\mathbf{r}$ directly from the four edges.

Tasks

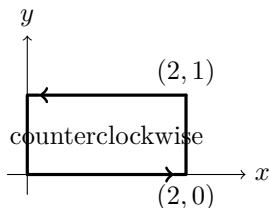
1. Compute each oriented edge contribution.
2. Add the four contributions.
3. Check which signs are forced by the traversal direction.

Question 8 – Solution

Strategy. Write $\mathbf{F} \cdot d\mathbf{r} = y^2 dx + x dy$ and exploit the constant coordinate on each edge.

Step 1: Bottom and right Along the bottom, $y = 0$ and $dy = 0$, so the contribution is 0. Along the right edge, $x = 2$ and $y : 0 \rightarrow 1$, giving

$$\int_0^1 2 dy = 2.$$



Step 2: Top and left Along the top, $y = 1$ and counterclockwise travel has $x : 2 \rightarrow 0$:

$$\int_2^0 1 dx = -2.$$

On the left, $x = 0$, and $dx = 0$, so the contribution is 0.

Step 3: Sum

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = 0 + 2 - 2 + 0 = 0.$$

Verification The right-edge contribution is positive because both the field's vertical component and dy are positive. The top contribution is negative because $y^2 > 0$ while $dx < 0$; their equal magnitudes cancel.