

Question 7

The field $\mathbf{F}(x, y) = \langle -y, x \rangle$ acts between $A = (0, 0)$ and $B = (1, 1)$. Compare the work along

$$C_1 : \mathbf{r}(t) = \langle t, t \rangle, \quad 0 \leq t \leq 1,$$

and the broken path C_2 that goes from $(0, 0)$ to $(1, 0)$ to $(1, 1)$.

Tasks

1. Compute the work along C_1 .
2. Compute the work along both pieces of C_2 .
3. Explain what the comparison demonstrates.

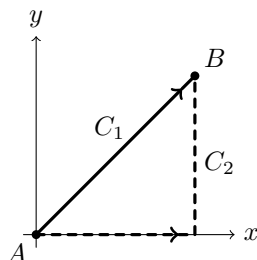
Question 7 – Solution

Strategy. Evaluate the same vector-field line integral on two explicitly parametrized paths with common endpoints.

Step 1: Diagonal path On C_1 ,

$$\mathbf{F}(\mathbf{r}(t)) = \langle -t, t \rangle, \quad \mathbf{r}'(t) = \langle 1, 1 \rangle.$$

The dot product is $-t + t = 0$, so $W_1 = \boxed{0}$.



Step 2: Broken path On the horizontal piece $\langle t, 0 \rangle$, the dot product $\langle 0, t \rangle \cdot \langle 1, 0 \rangle$ is zero. On the vertical piece $\langle 1, s \rangle$,

$$\langle -s, 1 \rangle \cdot \langle 0, 1 \rangle = 1, \quad 0 \leq s \leq 1.$$

Thus

$$W_2 = 0 + \int_0^1 1 \, ds = \boxed{1}.$$

Step 3: Compare Since $W_1 \neq W_2$ despite identical endpoints, the work for this field depends on the path taken.

Verification Along the diagonal the field is perpendicular to the motion. Along the final vertical piece of C_2 , its upward component is 1, explaining the extra unit of work.