

Question 6

For a constant a , let

$$\mathbf{F}_a(x, y) = \langle ay, x \rangle.$$

Along the parabola $\mathbf{r}(t) = \langle t, t^2 \rangle$, $0 \leq t \leq 1$, the work done is 2. Determine a .

Tasks

1. Express the work in terms of a .
2. Solve for a .
3. Substitute the result and verify the specified work.

Question 6 – Solution

Strategy. Restrict the one-parameter family of fields to the given curve; the work condition becomes a linear equation in a .

Step 1: Build the integrand

$$\mathbf{F}_a(\mathbf{r}(t)) = \langle at^2, t \rangle, \quad \mathbf{r}'(t) = \langle 1, 2t \rangle.$$

Thus

$$\mathbf{F}_a(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = at^2 + 2t^2 = (a + 2)t^2.$$

Step 2: Impose the work condition

$$2 = \int_0^1 (a + 2)t^2 dt = \frac{a + 2}{3}.$$

Therefore $a + 2 = 6$, and

$$\boxed{a = 4}.$$

Step 3: Verify With $a = 4$,

$$\int_0^1 (4 + 2)t^2 dt = 6 \left[\frac{t^3}{3} \right]_0^1 = 2,$$

which is the required work.

Uniqueness check The coefficient of a in the work is $\int_0^1 t^2 dt = 1/3 \neq 0$, so no second value of a can satisfy the condition.