

**Question 4**

Let  $C$  be the unit circle and let

$$\mathbf{F}(x, y) = \langle -y, x \rangle.$$

Compute  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  when  $C$  is traversed (a) counterclockwise and (b) clockwise.

**Tasks**

1. Parametrize the counterclockwise circle and evaluate the integral.
2. Use orientation reversal to obtain the clockwise value.
3. Interpret the signs using the direction of the field.

### Question 4 – Solution

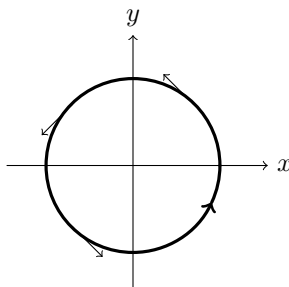
**Strategy.** On the unit circle this field is the counterclockwise unit tangent, making the dot product especially simple.

**Step 1: Counterclockwise traversal** Take

$$\mathbf{r}(t) = \langle \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi.$$

Then

$$\mathbf{F}(\mathbf{r}(t)) = \langle -\sin t, \cos t \rangle = \mathbf{r}'(t).$$



field and traversal are counterclockwise

Consequently,

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{r}'(t) \cdot \mathbf{r}'(t) dt = \int_0^{2\pi} 1 dt = \boxed{2\pi}.$$

**Step 2: Reverse the orientation** Reversing a vector-field line integral changes its sign, so the clockwise value is

$$\boxed{-2\pi}.$$

**Verification** The field assists counterclockwise motion at every point and opposes clockwise motion at every point, so the respective signs must be positive and negative.