

Question 3

Let C travel from $A = (0, 0)$ to $B = (2, 0)$ and then from B to $C = (2, 1)$. For

$$\mathbf{F}(x, y) = \langle -y, x \rangle,$$

evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Tasks

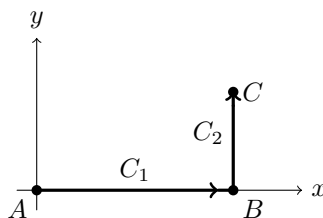
1. Parametrize each segment with the stated orientation.
2. Compute the contribution from each segment.
3. Explain geometrically why the first contribution vanishes.

Question 3 – Solution

Strategy. Split the piecewise path and add the two oriented line integrals.

Step 1: Horizontal segment Use $\mathbf{r}_1(t) = \langle t, 0 \rangle$, $0 \leq t \leq 2$. Then

$$\mathbf{F}(\mathbf{r}_1(t)) = \langle 0, t \rangle, \quad \mathbf{r}'_1(t) = \langle 1, 0 \rangle, \quad \mathbf{F} \cdot \mathbf{r}'_1 = 0.$$



The field is vertical along this segment while the motion is horizontal, so the vectors are perpendicular.

Step 2: Vertical segment Use $\mathbf{r}_2(s) = \langle 2, s \rangle$, $0 \leq s \leq 1$. Thus

$$\mathbf{F}(\mathbf{r}_2(s)) = \langle -s, 2 \rangle, \quad \mathbf{r}'_2(s) = \langle 0, 1 \rangle,$$

and

$$\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = \int_0^1 2 \, ds = 2.$$

Step 3: Add Hence

$$\boxed{\int_C \mathbf{F} \cdot d\mathbf{r} = 0 + 2 = 2}.$$

Verification Only the upward component on the vertical segment contributes, and that component is the constant 2 over a distance of 1.