

Question 2

A particle follows the helix

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi,$$

through the constant force field $\mathbf{F} = \langle 2, -1, 3 \rangle$.

Tasks

1. Compute the work done by the force.
2. Identify the contributions from the horizontal and vertical motion.
3. Check the result using the net displacement.

Question 2 – Solution

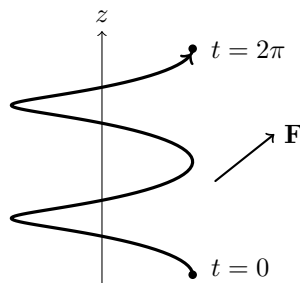
Strategy. Dot the constant force with the velocity, then use the displacement as an independent check.

Step 1: Velocity and dot product

$$\mathbf{r}'(t) = \langle -\sin t, \cos t, 1 \rangle,$$

so

$$\mathbf{F} \cdot \mathbf{r}'(t) = -2 \sin t - \cos t + 3.$$

**Step 2: Work**

$$\begin{aligned} W &= \int_0^{2\pi} (-2 \sin t - \cos t + 3) dt \\ &= 0 + 0 + 6\pi = \boxed{6\pi}. \end{aligned}$$

The horizontal coordinates complete a full turn, so their signed contributions cancel. The vertical rise is 2π , producing work $3(2\pi)$.

Verification The net displacement is $\mathbf{r}(2\pi) - \mathbf{r}(0) = \langle 0, 0, 2\pi \rangle$. For a constant force, its dot product with this displacement is $\langle 2, -1, 3 \rangle \cdot \langle 0, 0, 2\pi \rangle = 6\pi$.