

Question 1

Let

$$\mathbf{F}(x, y) = \langle y, x^2 \rangle, \quad \mathbf{r}(t) = \langle t, t^2 \rangle, \quad 0 \leq t \leq 1.$$

Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Tasks

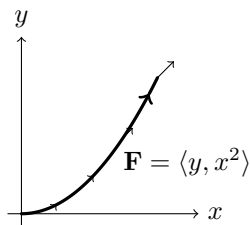
1. Evaluate $\mathbf{F}$ along the curve and compute $\mathbf{r}'(t)$.
2. Form the dot product and evaluate the integral.
3. State how the sign of the integrand agrees with the geometry.

Question 1 – Solution

Strategy. Use $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$.

Step 1: Restrict the field to the curve Since $x = t$ and $y = t^2$,

$$\mathbf{F}(\mathbf{r}(t)) = \langle t^2, t^2 \rangle, \quad \mathbf{r}'(t) = \langle 1, 2t \rangle.$$



Step 2: Form the scalar integrand

$$\mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) = t^2 + 2t^3.$$

Therefore

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 (t^2 + 2t^3) dt = \left[\frac{t^3}{3} + \frac{t^4}{2} \right]_0^1 = \boxed{\frac{5}{6}}.$$

Verification For $0 \leq t \leq 1$, both components of the field and both components of the velocity are nonnegative. Their dot product must be nonnegative, consistent with the positive answer.