

Question 10

Two curves connect $A = (0, 0)$ to $B = (1, 1)$:

$$C_1 : y = x, \quad C_2 : y = x^2, \quad 0 \leq x \leq 1.$$

Compare

$$\int_C y \, dx$$

along the two paths.

Tasks

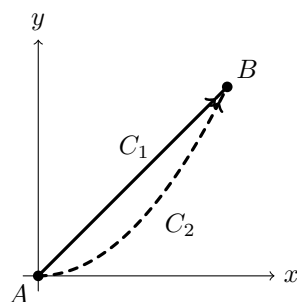
1. Evaluate the integral on C_1 .
2. Evaluate it on C_2 .
3. Explain what the unequal values demonstrate about path dependence.

Question 10 – Solution

Strategy. Use the shared coordinate parameter x on both paths, then compare the scalar functions traced between the same endpoints.

Step 1: Straight path On C_1 , $y = x$, so

$$\int_{C_1} y \, dx = \int_0^1 x \, dx = \boxed{\frac{1}{2}}.$$



Step 2: Parabolic path On C_2 , $y = x^2$, so

$$\int_{C_2} y \, dx = \int_0^1 x^2 \, dx = \boxed{\frac{1}{3}}.$$

Step 3: Interpretation Both curves have the same initial point, terminal point, and orientation, yet

$$\frac{1}{2} \neq \frac{1}{3}.$$

Therefore the value of $\int_C y \, dx$ is not determined by endpoints alone; it depends on the path between them.

Verification For $0 < x < 1$, the line $y = x$ lies above the parabola $y = x^2$. With $dx > 0$, its integral must be larger, consistent with $1/2 > 1/3$.