

Question 9

On the parabola $C : y = x^2$ from $(0, 0)$ to $(1, 1)$, a constant a satisfies

$$\int_C ax \, dx + y \, dy = 3.$$

Determine a .

Tasks

1. Convert the entire integral to the parameter x .
2. Solve the resulting equation for a .
3. Substitute the result to verify the specified value.

Question 9 – Solution

Strategy. Use $dy = 2x \, dx$ to make the unknown coefficient appear in a simple algebraic equation.

Step 1: Convert Since $y = x^2$ and $dy = 2x \, dx$,

$$\int_C ax \, dx + y \, dy = \int_0^1 (ax + x^2(2x)) \, dx.$$

Therefore

$$\int_0^1 (ax + 2x^3) \, dx = \frac{a}{2} + \frac{1}{2}.$$

Step 2: Solve

$$\frac{a}{2} + \frac{1}{2} = 3, \quad \frac{a}{2} = \frac{5}{2}, \quad \boxed{a = 5}.$$

Step 3: Verify With $a = 5$,

$$\int_0^1 (5x + 2x^3) \, dx = \left[\frac{5x^2}{2} + \frac{x^4}{2} \right]_0^1 = \frac{5}{2} + \frac{1}{2} = 3.$$

Verification The computed value satisfies the original integral exactly. Because the coefficient of a is $\int_0^1 x \, dx = 1/2 \neq 0$, the solution is unique.