

Question 8

Let

$$C : \mathbf{r}(t) = \langle \cos t, \sin t, t \rangle, \quad 0 \leq t \leq \frac{\pi}{2}.$$

Evaluate

$$\int_C z \, dx + x \, dy + y \, dz.$$

Tasks

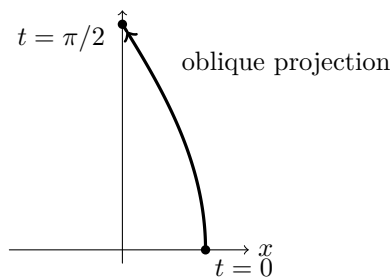
1. Compute dx , dy , and dz .
2. Convert the line integral to one parameter and evaluate it.
3. Verify the cancellation among the three contributions.

Question 8 – Solution

Strategy. Treat each coordinate differential separately, then combine all three ordinary integrals.

Step 1: Differential data

$$\begin{aligned}x &= \cos t, & y &= \sin t, & z &= t, \\dx &= -\sin t \, dt, & dy &= \cos t \, dt, & dz &= dt.\end{aligned}$$



Step 2: Evaluate

$$\int_C z \, dx + x \, dy + y \, dz = \int_0^{\pi/2} (-t \sin t + \cos^2 t + \sin t) \, dt.$$

The three contributions are

$$\int_0^{\pi/2} -t \sin t \, dt = -1, \quad \int_0^{\pi/2} \cos^2 t \, dt = \frac{\pi}{4}, \quad \int_0^{\pi/2} \sin t \, dt = 1.$$

Hence

$$\boxed{\int_C z \, dx + x \, dy + y \, dz = \frac{\pi}{4}}.$$

Verification The first and third contributions cancel exactly, leaving only $\int_0^{\pi/2} \cos^2 t \, dt = \pi/4$.