

**Question 7**

Let  $C$  be the vertical segment  $x = 2$  from  $(2, 0)$  to  $(2, 3)$ . Evaluate both

$$\int_C (x + y) \, dx \quad \text{and} \quad \int_C (x + y) \, dy.$$

**Tasks**

1. Parametrize the vertical segment.
2. Evaluate both integrals.
3. Explain why treating  $y$  as a function of  $x$  is inappropriate here.

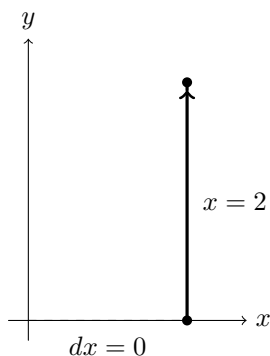
**Question 7 – Solution**

**Strategy.** Parametrize by the changing coordinate  $y$ ; the constant coordinate has zero differential.

**Step 1: Parametrize**

$$\mathbf{r}(t) = \langle 2, t \rangle, \quad 0 \leq t \leq 3.$$

Thus  $dx = 0$ ,  $dy = dt$ , and  $x + y = 2 + t$ .

**Step 2: Evaluate**

$$\boxed{\int_C (x + y) dx = 0}$$

because  $dx = 0$  everywhere. Meanwhile,

$$\int_C (x + y) dy = \int_0^3 (2 + t) dt = \left[ 2t + \frac{t^2}{2} \right]_0^3 = \boxed{\frac{21}{2}}.$$

**Step 3: Representation issue** A vertical segment assigns many  $y$ -values to the single value  $x = 2$ , so it cannot be written as a single-valued graph  $y = g(x)$ . Parametrization avoids this limitation.

**Verification** The second integrand rises linearly from 2 to 5; its average  $7/2$  times the  $y$ -interval length 3 gives  $21/2$ .