

Question 6

Let C be the graph $y = x^3$ from $(0, 0)$ to $(1, 1)$. Evaluate

$$\int_C x^2 dy.$$

Tasks

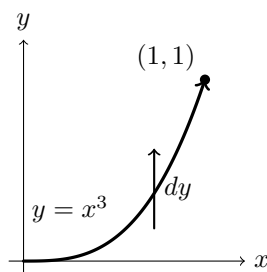
1. Convert dy using the graph equation.
2. Evaluate the resulting single-variable integral.
3. Verify the result by using y as the parameter instead.

Question 6 – Solution

Strategy. First calculate with x as parameter, then reparametrize by y to confirm invariance under a valid change of parameter.

Step 1: Use x

$$y = x^3, \quad dy = 3x^2 dx, \quad 0 \leq x \leq 1.$$



Therefore

$$\int_C x^2 dy = 3 \int_0^1 x^4 dx = 3 \left[\frac{x^5}{5} \right]_0^1 = \boxed{\frac{3}{5}}.$$

Step 2: Use y instead On this interval, $x = y^{1/3}$, so $x^2 = y^{2/3}$. Since the differential is already dy ,

$$\int_C x^2 dy = \int_0^1 y^{2/3} dy = \left[\frac{3}{5} y^{5/3} \right]_0^1 = \frac{3}{5}.$$

Verification Both parametrizations preserve the motion from $(0,0)$ to $(1,1)$ and give the same value, confirming the differential conversion.