

**Question 5**

Let  $C$  be the boundary of the rectangle

$$0 \leq x \leq 3, \quad 0 \leq y \leq 2,$$

traversed counterclockwise. Evaluate

$$\oint_C y \, dx$$

by splitting the boundary into four segments.

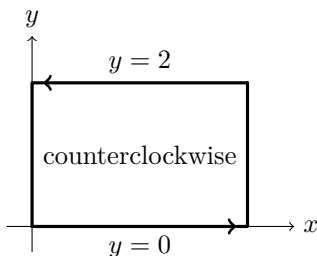
**Tasks**

1. Determine which edges contribute zero.
2. Evaluate the contribution from each horizontal edge.
3. Add the pieces and check the sign from the orientation.

**Question 5 – Solution**

**Strategy.** On vertical edges  $dx = 0$ , while  $y$  is constant on each horizontal edge.

**Step 1: Traverse the edges**



Along the bottom edge,  $y = 0$ , so its contribution is 0. On the right and left edges,  $x$  is constant, so  $dx = 0$  and both contributions are also 0.

**Step 2: Top edge** Counterclockwise travel moves along the top from  $(3, 2)$  to  $(0, 2)$ . Thus

$$\int_{\text{top}} y \, dx = \int_3^0 2 \, dx = 2[x]_3^0 = -6.$$

**Step 3: Sum**

$$\oint_C y \, dx = 0 + 0 - 6 + 0 = -6.$$

**Verification** The only nonzero edge has  $y > 0$  and moves left, so  $dx < 0$ ; a negative answer is necessary. Reversing the rectangle would change the answer to  $+6$ .