

Question 4

Let C_1 be the upper unit semicircle oriented from $(1, 0)$ to $(-1, 0)$, and let C_2 be the same arc with the reverse orientation. Compute

$$\int_C y \, dx$$

for both orientations.

Tasks

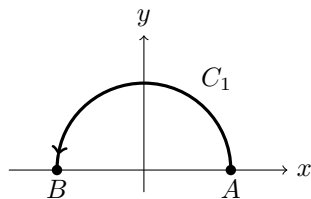
1. Parametrize C_1 and evaluate the integral.
2. Obtain the value on C_2 .
3. Explain why orientation matters for dx but not for ds .

Question 4 – Solution

Strategy. Compute once in the counterclockwise direction, then use the sign change caused by reversing dx .

Step 1: Forward orientation For C_1 , use

$$x = \cos \theta, \quad y = \sin \theta, \quad 0 \leq \theta \leq \pi, \quad dx = -\sin \theta \, d\theta.$$



reversal changes the sign

Therefore

$$\int_{C_1} y \, dx = - \int_0^\pi \sin^2 \theta \, d\theta = \boxed{-\frac{\pi}{2}}.$$

Step 2: Reverse orientation Reversing the same curve negates the coordinate differential, so

$$\boxed{\int_{C_2} y \, dx = \frac{\pi}{2}}.$$

Step 3: Why the rules differ Under reversal, $dx = x'(t)dt$ changes sign because the derivative reverses. By contrast, $ds = |\mathbf{r}'(t)|dt$ uses a magnitude and remains nonnegative.

Verification On C_1 , $y \geq 0$ while x decreases, so $dx \leq 0$ and the integral must be nonpositive. Its reverse must be nonnegative, matching the two answers.