

Question 3

Let C be the straight segment from $A = (0, 0)$ to $B = (2, 1)$. Evaluate

$$\int_C (x + y) dx + (x - y) dy.$$

Tasks

1. Parametrize the segment with the stated orientation.
2. Convert both differentials to the parameter.
3. Evaluate and check the endpoint orientation.

Question 3 – Solution

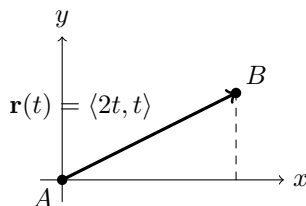
Strategy. A combined differential-form integral becomes one ordinary integral after substituting $dx = x'(t)dt$ and $dy = y'(t)dt$.

Step 1: Parameter data

$$\mathbf{r}(t) = \langle 2t, t \rangle, \quad 0 \leq t \leq 1,$$

so

$$x + y = 3t, \quad x - y = t, \quad dx = 2 dt, \quad dy = dt.$$



Step 2: Evaluate

$$\begin{aligned} \int_C (x + y) dx + (x - y) dy &= \int_0^1 [(3t)(2) + (t)(1)] dt \\ &= \int_0^1 7t dt = \boxed{\frac{7}{2}}. \end{aligned}$$

Verification At $t = 0$ the parametrization gives A , and at $t = 1$ it gives B , so the orientation is correct. Directly grouping the transformed integrand gives $[2(x + y) + (x - y)]dt = (3x + y)dt = 7t dt$, confirming the algebra.