

Question 2

On the same parabola $C : y = x^2$ from $(0, 0)$ to $(2, 4)$, evaluate

$$\int_C x \, dy.$$

Tasks

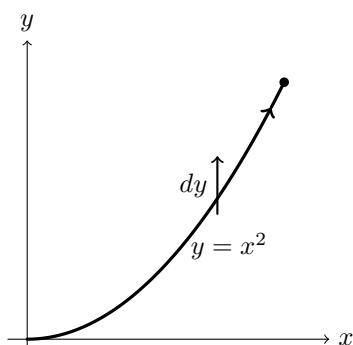
1. Express dy in terms of dx .
2. Evaluate the integral.
3. Compare the answer with $\int_C y \, dx = 8/3$ and explain why they differ.

Question 2 – Solution

Strategy. Use the graph relation to write $dy = y'(x) dx$.

Step 1: Convert the differential Along $y = x^2$,

$$dy = 2x dx, \quad 0 \leq x \leq 2.$$



Step 2: Evaluate

$$\int_C x dy = \int_0^2 x(2x) dx = 2 \left[\frac{x^3}{3} \right]_0^2 = \boxed{\frac{16}{3}}.$$

Step 3: Compare Although both integrals use the same curve, they measure change in different coordinates:

$$y dx = x^2 dx, \quad x dy = x(2x dx) = 2x^2 dx.$$

The second integrand is exactly twice the first, so its value is twice $8/3$.

Verification Here y increases from 0 to 4 and $0 \leq x \leq 2$, so the value must lie between 0 and $2(4) = 8$. The result $16/3$ does.