

Question 1

Let C be the parabola $y = x^2$ from $(0, 0)$ to $(2, 4)$. Evaluate

$$\int_C y \, dx.$$

Tasks

1. Parametrize the curve using x as the parameter.
2. Replace dx correctly and evaluate the integral.
3. Explain why no arc-length factor appears.

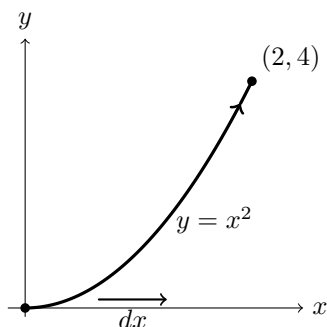
Question 1 – Solution

Strategy. For an integral with respect to x , use $dx = x'(t) dt$ rather than the arc-length element.

Step 1: Parametrize

$$\mathbf{r}(t) = \langle t, t^2 \rangle, \quad 0 \leq t \leq 2.$$

Thus $x = t$, $y = t^2$, and $dx = dt$.



Step 2: Evaluate

$$\int_C y \, dx = \int_0^2 t^2 \, dt = \left[\frac{t^3}{3} \right]_0^2 = \boxed{\frac{8}{3}}.$$

Step 3: Distinguish from ds The differential is $dx = x'(t) dt = dt$. The speed $\sqrt{1 + 4t^2}$ would be used only for an integral with respect to ds ; inserting it here would change the problem.

Verification Since x increases from 0 to 2 and $0 \leq y \leq 4$, the integral must lie between 0 and $4(2) = 8$. The value $8/3$ satisfies this bound.