

Question 9

For $a > 0$, a straight wire runs from $(0, 0)$ to (a, a) and has density

$$\delta(x, y) = x + y.$$

Its mass is $8\sqrt{2}$. Determine a and the wire's length.

Tasks

1. Parametrize the wire and derive its mass as a function of a .
2. Solve the inverse condition, respecting $a > 0$.
3. Compute the resulting length and verify the mass.

Question 9 – Solution

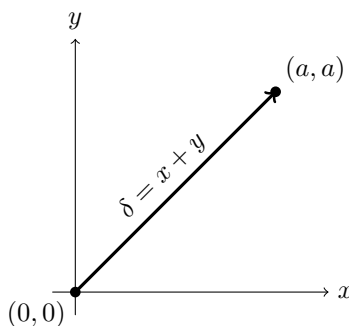
Strategy. Express the line integral entirely in terms of the unknown endpoint coordinate a , then solve the resulting quadratic equation.

Step 1: Mass formula Use

$$\mathbf{r}(t) = \langle at, at \rangle, \quad 0 \leq t \leq 1.$$

Then $\delta = 2at$ and $ds = |\langle a, a \rangle| dt = a\sqrt{2} dt$. Therefore

$$M(a) = \int_0^1 (2at)(a\sqrt{2}) dt = 2a^2\sqrt{2} \left[\frac{t^2}{2} \right]_0^1 = a^2\sqrt{2}.$$



Step 2: Recover a The given mass implies

$$a^2\sqrt{2} = 8\sqrt{2}, \quad a^2 = 8.$$

Because $a > 0$,

$$\boxed{a = 2\sqrt{2}}.$$

Step 3: Length and verification The diagonal length is

$$L = \sqrt{a^2 + a^2} = a\sqrt{2} = (2\sqrt{2})(\sqrt{2}) = \boxed{4}.$$

Substitution into the mass formula gives $M = (2\sqrt{2})^2\sqrt{2} = 8\sqrt{2}$, exactly the specified mass.