

Question 6

Let C be the unit circle. Compute

$$\int_C (x + 2) \, ds$$

once with a counterclockwise parametrization and once with a clockwise parametrization.

Tasks

1. Write parametrizations for both orientations.
2. Show directly that both integrals have the same value.
3. Explain why every scalar line integral with respect to ds is orientation-independent.

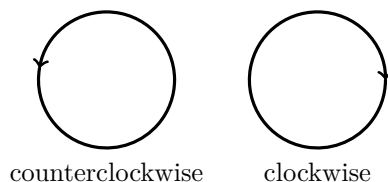
Question 6 – Solution

Strategy. Reversing direction changes the parameter derivative's sign but not its magnitude, so ds is unchanged.

Step 1: Counterclockwise Let $\mathbf{r}_+(t) = \langle \cos t, \sin t \rangle$, $0 \leq t \leq 2\pi$. Since $|\mathbf{r}'_+(t)| = 1$,

$$I_+ = \int_0^{2\pi} (\cos t + 2) dt = 0 + 4\pi = 4\pi.$$

same unit circle, opposite orientations



Step 2: Clockwise Let $\mathbf{r}_-(t) = \langle \cos t, -\sin t \rangle$, $0 \leq t \leq 2\pi$. Again the speed is 1, and $x = \cos t$, so

$$I_- = \int_0^{2\pi} (\cos t + 2) dt = \boxed{4\pi}.$$

Step 3: General reason Under a reversed parametrization, $\tilde{\mathbf{r}}(t) = \mathbf{r}(a+b-t)$, the derivative is $-\mathbf{r}'(a+b-t)$, but

$$|\tilde{\mathbf{r}}'(t)| = |\mathbf{r}'(a+b-t)|.$$

After substitution, the scalar values and arc-length factors are identical. Therefore $\int_C f ds$ does not depend on orientation.

Verification Here the x -term cancels by symmetry, while the constant 2 contributes 2 times the circumference 2π , confirming 4π .