

Question 5

The piecewise smooth curve $C = C_1 \cup C_2$ travels from

$$A = (0, 0) \quad \text{to} \quad B = (1, 0) \quad \text{to} \quad D = (1, 2)$$

along straight segments. Evaluate

$$\int_C (xy + 1) \, ds.$$

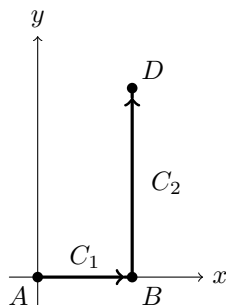
Tasks

1. Parametrize each segment.
2. Evaluate the integral on each piece and add the results.
3. Check the contribution using the integrand along each segment.

Question 5 – Solution

Strategy. A scalar line integral over a piecewise curve is the sum of the integrals over its smooth pieces.

Step 1: First segment



For C_1 , use $\mathbf{r}_1(t) = \langle t, 0 \rangle$, $0 \leq t \leq 1$. Then $ds = dt$ and $xy + 1 = 1$, so

$$\int_{C_1} (xy + 1) ds = \int_0^1 1 dt = 1.$$

Step 2: Second segment For C_2 , use $\mathbf{r}_2(u) = \langle 1, u \rangle$, $0 \leq u \leq 2$. Then $ds = du$ and $xy + 1 = u + 1$, giving

$$\int_{C_2} (xy + 1) ds = \int_0^2 (u + 1) du = \left[\frac{u^2}{2} + u \right]_0^2 = 4.$$

Hence

$$\boxed{\int_C (xy + 1) ds = 1 + 4 = 5}.$$

Verification On C_1 the constant value 1 times length 1 gives 1. On C_2 the integrand rises linearly from 1 to 3, so its average 2 times length 2 gives 4.