

**Question 4**

Let  $C$  be the helix

$$\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, t \rangle, \quad 0 \leq t \leq \pi.$$

Evaluate

$$\int_C (x^2 + y^2 + z) \, ds.$$

**Tasks**

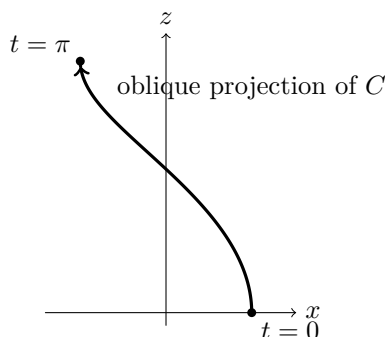
1. Express the scalar function along the helix.
2. Compute the speed and evaluate the integral.
3. Verify the result by separating the constant and variable parts.

## Question 4 – Solution

**Strategy.** Substitute the parametrization into both the scalar function and the arc-length factor.

**Step 1: Restrict the integrand** Along  $C$ ,

$$x^2 + y^2 + z = 4 \cos^2 t + 4 \sin^2 t + t = 4 + t.$$



**Step 2: Arc length and integral**

$$\mathbf{r}'(t) = \langle -2 \sin t, 2 \cos t, 1 \rangle, \quad |\mathbf{r}'(t)| = \sqrt{4 + 1} = \sqrt{5}.$$

Therefore

$$\begin{aligned} \int_C (x^2 + y^2 + z) \, ds &= \sqrt{5} \int_0^\pi (4 + t) \, dt \\ &= \sqrt{5} \left[ 4t + \frac{t^2}{2} \right]_0^\pi = \boxed{\sqrt{5} \left( 4\pi + \frac{\pi^2}{2} \right)}. \end{aligned}$$

**Verification** The constant part 4 contributes  $4L = 4\pi\sqrt{5}$  because  $L = \pi\sqrt{5}$ . The  $z = t$  part has average  $\pi/2$  and contributes  $(\pi/2)L = \pi^2\sqrt{5}/2$ , matching the two terms above.