

Question 3

Let C be the upper semicircle

$$x^2 + y^2 = 4, \quad y \geq 0,$$

oriented from $(2, 0)$ to $(-2, 0)$. Evaluate

$$\int_C (x^2 + y^2) \, ds.$$

Tasks

1. Parametrize the semicircle with its stated orientation.
2. Evaluate the line integral.
3. Give a geometric verification without integration.

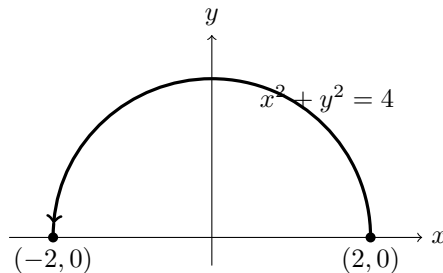
Question 3 – Solution

Strategy. On a circle, the scalar integrand $x^2 + y^2$ is constant, so the integral is that constant times arc length.

Step 1: Parametrization

$$\mathbf{r}(\theta) = \langle 2 \cos \theta, 2 \sin \theta \rangle, \quad 0 \leq \theta \leq \pi.$$

This begins at $(2, 0)$ and ends at $(-2, 0)$, as required.

**Step 2: Evaluate** Along C ,

$$x^2 + y^2 = 4, \quad |\mathbf{r}'(\theta)| = | \langle -2 \sin \theta, 2 \cos \theta \rangle | = 2.$$

Thus

$$\int_C (x^2 + y^2) ds = \int_0^\pi (4)(2) d\theta = \boxed{8\pi}.$$

Verification The upper semicircle has length $\frac{1}{2}(2\pi \cdot 2) = 2\pi$. Since the integrand is constantly 4, the integral must be $4(2\pi) = 8\pi$.