

Question 2

Evaluate

$$\int_C x \, ds,$$

where C is the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

Tasks

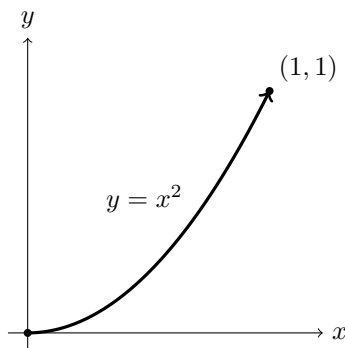
1. Use x itself as a parameter.
2. Derive the arc-length element and evaluate the integral.
3. Check the sign and size of the result.

Question 2 – Solution

Strategy. Parametrize the graph by $\mathbf{r}(x) = \langle x, x^2 \rangle$ so the given scalar function is already the parameter.

Step 1: Parameter and speed

$$\begin{aligned}\mathbf{r}(x) &= \langle x, x^2 \rangle, & 0 \leq x \leq 1, \\ \mathbf{r}'(x) &= \langle 1, 2x \rangle, & ds = \sqrt{1 + 4x^2} \, dx.\end{aligned}$$



Step 2: Evaluate Let $u = 1 + 4x^2$, so $du = 8x \, dx$. Then

$$\begin{aligned}\int_C x \, ds &= \int_0^1 x \sqrt{1 + 4x^2} \, dx \\ &= \frac{1}{8} \int_1^5 u^{1/2} \, du = \frac{1}{12} \left[u^{3/2} \right]_1^5 = \boxed{\frac{5\sqrt{5} - 1}{12}}.\end{aligned}$$

Verification Both x and ds are nonnegative, so the answer must be positive. Also $0 \leq x \leq 1$ implies the integral is no larger than the curve length; numerically the result is about 0.848, consistent with that bound.