

Question 1

Evaluate the scalar line integral

$$\int_C (x + y) \, ds,$$

where C is the line segment from $(0, 0)$ to $(3, 4)$.

Tasks

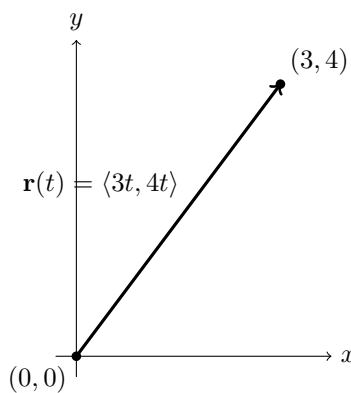
1. Parametrize C on $0 \leq t \leq 1$.
2. Compute ds and evaluate the integral.
3. Verify the constant speed and total length of the segment.

Question 1 – Solution

Strategy. Use linear interpolation between the endpoints, then replace ds by $|\mathbf{r}'(t)| dt$.

Step 1: Parametrize the segment

$$\mathbf{r}(t) = \langle 3t, 4t \rangle, \quad 0 \leq t \leq 1.$$



Along the curve,

$$x + y = 3t + 4t = 7t.$$

Step 2: Arc-length element

$$\mathbf{r}'(t) = \langle 3, 4 \rangle, \quad |\mathbf{r}'(t)| = 5, \quad ds = 5 dt.$$

Therefore

$$\int_C (x + y) ds = \int_0^1 (7t)(5) dt = 35 \left[\frac{t^2}{2} \right]_0^1 = \boxed{\frac{35}{2}}.$$

Verification The speed is constant and the parameter interval has length 1, so the curve length is 5, agreeing with the distance formula $\sqrt{3^2 + 4^2} = 5$. The integrand rises linearly from 0 to 7, whose average is 7/2; multiplying by length 5 again gives 35/2.