

Question 10

On the unit circle $x^2 + y^2 = 1$, consider

$$\mathbf{F}(x, y) = \langle 2x - y, x + 2y \rangle.$$

Tasks

1. Decompose $\mathbf{F}$ into outward radial and counterclockwise tangential components.
2. Find its magnitude and its angle from the outward radial direction.
3. Evaluate the decomposition at $(0, 1)$ and determine whether the field has any zero away from the origin.

Question 10 – Solution

Strategy. Recognize the field as a fixed combination of the radial vector and its 90° counterclockwise rotation.

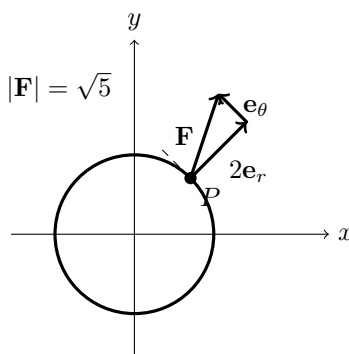
Step 1: Decompose On the unit circle,

$$\mathbf{e}_r = \langle x, y \rangle, \quad \mathbf{e}_\theta = \langle -y, x \rangle.$$

Therefore

$$\mathbf{F} = 2\langle x, y \rangle + \langle -y, x \rangle = \boxed{2\mathbf{e}_r + \mathbf{e}_\theta}.$$

The radial scalar component is 2 and the counterclockwise tangential scalar component is 1.



Step 2: Magnitude and angle Because the two unit vectors are perpendicular,

$$\boxed{|\mathbf{F}| = \sqrt{2^2 + 1^2} = \sqrt{5}}.$$

If α is measured counterclockwise from $\mathbf{e}_r$, then

$$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\right)}.$$

Step 3: Point check and zeros At $(0, 1)$,

$$\mathbf{e}_r = \langle 0, 1 \rangle, \quad \mathbf{e}_\theta = \langle -1, 0 \rangle, \quad \mathbf{F} = \boxed{\langle -1, 2 \rangle}.$$

For the full plane, $\mathbf{F} = \mathbf{0}$ gives $2x - y = 0$ and $x + 2y = 0$. Their coefficient determinant is $5 \neq 0$, so only the origin is a zero; in particular, there are none on the unit circle.

Verification The sample vector $\langle -1, 2 \rangle$ has length $\sqrt{5}$ and equals $2\mathbf{e}_r + \mathbf{e}_\theta$, confirming all parts.