

Question 9

Let

$$\mathbf{F}(x, y, z) = \langle y, z, x \rangle$$

and let the helix be

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle.$$

Tasks

1. Express the field along the helix as $\mathbf{F}(\mathbf{r}(t))$.
2. Find the unit tangent $\mathbf{T}(t)$.
3. At $t = 0$, decompose $\mathbf{F}(\mathbf{r}(0))$ into components parallel and perpendicular to the helix.

Question 9 – Solution

Strategy. Substitute the curve into the field, then use vector projection onto the unit tangent.

Step 1: Restrict the field to the curve

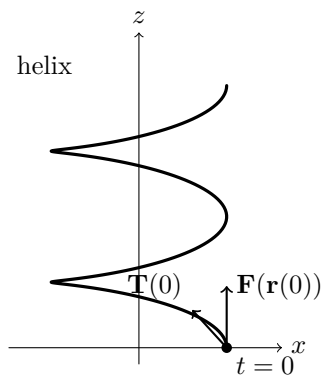
$$\mathbf{F}(\mathbf{r}(t)) = \langle \sin t, t, \cos t \rangle.$$

Also,

$$\mathbf{r}'(t) = \langle -\sin t, \cos t, 1 \rangle, \quad |\mathbf{r}'(t)| = \sqrt{2},$$

so

$$\mathbf{T}(t) = \frac{1}{\sqrt{2}} \langle -\sin t, \cos t, 1 \rangle.$$



Step 2: Data at $t = 0$

$$\mathbf{F}(\mathbf{r}(0)) = \langle 0, 0, 1 \rangle, \quad \mathbf{T}(0) = \frac{1}{\sqrt{2}} \langle 0, 1, 1 \rangle.$$

The scalar tangential component is

$$\mathbf{F} \cdot \mathbf{T} = \frac{1}{\sqrt{2}}.$$

Step 3: Vector decomposition

$$\mathbf{F}_{\parallel} = (\mathbf{F} \cdot \mathbf{T})\mathbf{T} = \left\langle 0, \frac{1}{2}, \frac{1}{2} \right\rangle,$$

$$\mathbf{F}_{\perp} = \mathbf{F} - \mathbf{F}_{\parallel} = \left\langle 0, -\frac{1}{2}, \frac{1}{2} \right\rangle.$$

Verification The two boxed vectors sum to $\langle 0, 0, 1 \rangle$, and $\mathbf{F}_{\perp} \cdot \mathbf{T}(0) = 0$.