

Question 8

For a real parameter λ , consider

$$\mathbf{F}_\lambda(x, y) = \langle x - y, \lambda x + y \rangle.$$

Tasks

1. Find all equilibrium points satisfying $\mathbf{F}_\lambda = \mathbf{0}$.
2. Identify the exceptional value of λ and describe its equilibrium set.
3. For that exceptional value, describe the field on either side of the equilibrium set.

Question 8 – Solution

Strategy. Solve the two component equations and separate the parameter value that makes them dependent.

Step 1: Solve generally The first component gives $x = y$. Substituting into the second gives

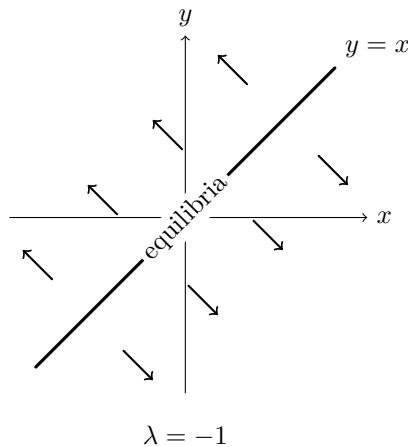
$$\lambda x + y = (\lambda + 1)x = 0.$$

If $\lambda \neq -1$, then $x = 0$ and $y = 0$, so

$$\boxed{\lambda \neq -1 : \text{the origin is the only equilibrium}}.$$

Step 2: Exceptional value If $\lambda = -1$, the second component is $-x + y$, the negative of the first. Every point with $x = y$ is then a zero:

$$\boxed{\lambda = -1 : \{(x, y) : y = x\} \text{ is the equilibrium set}}.$$



Step 3: Behavior off the line For $\lambda = -1$, set $d = x - y$. Then

$$\mathbf{F}_{-1} = \langle d, -d \rangle = d\langle 1, -1 \rangle.$$

This vector is normal to $y = x$. When $x > y$, it points southeast; when $x < y$, it points northwest. Both directions lead away from the equilibrium line.

Verification The coefficient determinant is $1(1) - (-1)\lambda = 1 + \lambda$, which vanishes exactly at $\lambda = -1$, confirming the case split.