

Question 7

A linear planar vector field has the form

$$\mathbf{F}(x, y) = \langle ax + by, cx + dy \rangle.$$

Measurements give

$$\mathbf{F}(1, 0) = \langle 2, -1 \rangle, \quad \mathbf{F}(0, 1) = \langle 3, 4 \rangle.$$

Tasks

1. Determine a, b, c, d and write the field explicitly.
2. Evaluate $\mathbf{F}(-1, 2)$.
3. Find every point where $\mathbf{F} = \mathbf{0}$ and explain uniqueness.

Question 7 – Solution

Strategy. The values on the coordinate basis vectors are exactly the columns of the matrix representing a linear field.

Step 1: Recover the coefficients Substitution at $(1, 0)$ gives $\langle a, c \rangle = \langle 2, -1 \rangle$, while substitution at $(0, 1)$ gives $\langle b, d \rangle = \langle 3, 4 \rangle$. Hence

$$\boxed{\mathbf{F}(x, y) = \langle 2x + 3y, -x + 4y \rangle}.$$

Equivalently,

$$\mathbf{F}(x, y) = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Step 2: Evaluate

$$\mathbf{F}(-1, 2) = \langle -2 + 6, 1 + 8 \rangle = \boxed{\langle 4, 9 \rangle}.$$

Step 3: Find zero vectors Solve

$$2x + 3y = 0, \quad -x + 4y = 0.$$

The second equation gives $x = 4y$; the first then gives $11y = 0$. Thus

$$\boxed{(x, y) = (0, 0)}$$

is the only zero. Equivalently, the coefficient determinant is $2(4) - 3(-1) = 11 \neq 0$, so the linear map has a trivial nullspace.

Verification The recovered formula returns both measured vectors when the two basis points are substituted, so all four coefficients satisfy the data.