

Question 6

A temperature distribution is

$$T(x, y) = 100 - x^2 - 4y^2.$$

The heat-flux vector field is $\mathbf{H} = -\frac{1}{2}\nabla T$.

Tasks

1. Find a formula for $\mathbf{H}(x, y)$.
2. Evaluate its magnitude and unit direction at $P = (2, 1)$.
3. Show that the heat flux is normal to the isotherm through P and identify that isotherm.

Question 6 – Solution

Strategy. The gradient is normal to level curves; multiplying it by a negative constant reverses direction but preserves normality.

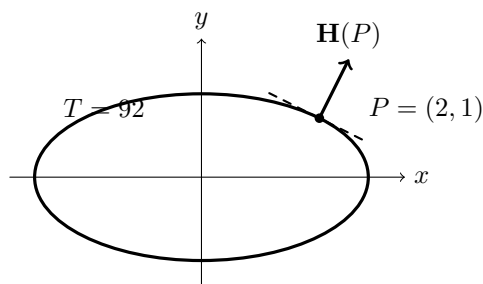
Step 1: Construct the field

$$\nabla T = \langle -2x, -8y \rangle, \quad \mathbf{H} = -\frac{1}{2}\nabla T = \boxed{\langle x, 4y \rangle}.$$

Step 2: Evaluate at P

$$\mathbf{H}(2, 1) = \langle 2, 4 \rangle, \quad |\mathbf{H}(2, 1)| = \boxed{2\sqrt{5}},$$

$$\widehat{\mathbf{H}}(2, 1) = \boxed{\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle}.$$



heat flux is normal to the isotherm

Step 3: Isotherm and normality Since $T(2, 1) = 100 - 4 - 4 = 92$, the isotherm is

$$x^2 + 4y^2 = 8.$$

Its normal direction is $\nabla(x^2 + 4y^2) = \langle 2x, 8y \rangle$. At P this is $\langle 4, 8 \rangle = 2\mathbf{H}(P)$, so the flux is normal to the ellipse and points toward lower temperature.

Verification Moving a short distance along $\mathbf{H}(P)$ increases $x^2 + 4y^2$, so it decreases $T = 100 - (x^2 + 4y^2)$, as heat flow should.