

Question 5

Let

$$\mathbf{F}(x, y) = \langle -y, x \rangle.$$

Tasks

1. Prove that $\mathbf{F}$ is tangent to every circle $x^2 + y^2 = c^2$ centered at the origin.
2. Find its magnitude and orientation.
3. At $P = (3, 4)$, find the vector and the corresponding unit tangent direction.

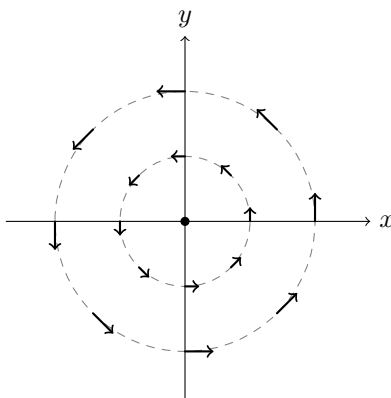
Question 5 – Solution

Strategy. A circle's radial vector is normal to the circle, so orthogonality to that vector proves tangency.

Step 1: Tangency At (x, y) the radial normal is $\mathbf{r} = \langle x, y \rangle$. Since

$$\mathbf{F} \cdot \mathbf{r} = \langle -y, x \rangle \cdot \langle x, y \rangle = -xy + xy = 0,$$

$\mathbf{F}$ is tangent to every centered circle.



vectors are tangent to centered circles

Step 2: Magnitude and orientation

$$|\mathbf{F}| = \sqrt{(-y)^2 + x^2} = \sqrt{x^2 + y^2} = r.$$

At $(r, 0)$ the vector is $\langle 0, r \rangle$, which points upward; therefore the orientation is counterclockwise.

Step 3: The point $(3, 4)$

$$\mathbf{F}(3, 4) = \langle -4, 3 \rangle, \quad |\mathbf{F}(3, 4)| = 5,$$

so the counterclockwise unit tangent is

$$\left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle.$$

Verification The dot product $\langle -4, 3 \rangle \cdot \langle 3, 4 \rangle = -12 + 12 = 0$, confirming tangency at P .