

Question 4

A three-dimensional gravitational field is modeled by

$$\mathbf{G}(x, y, z) = -12 \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}.$$

Tasks

1. Evaluate $\mathbf{G}$ at $P = (2, -1, 2)$.
2. Find its magnitude and a unit vector in its direction.
3. Explain the minus sign and state the field's domain.

Question 4 – Solution

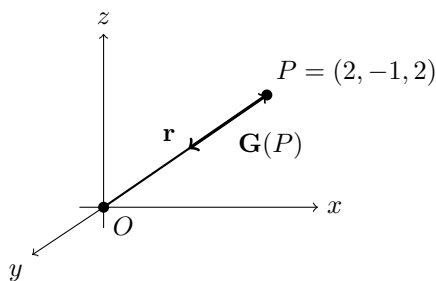
Strategy. Compute the distance from the origin first; it controls both the denominator and the inverse-square magnitude.

Step 1: Evaluate at the point At $P = (2, -1, 2)$,

$$r = \sqrt{2^2 + (-1)^2 + 2^2} = 3, \quad r^3 = 27.$$

Therefore

$$\mathbf{G}(P) = -\frac{12}{27}\langle 2, -1, 2 \rangle = \boxed{\left\langle -\frac{8}{9}, \frac{4}{9}, -\frac{8}{9} \right\rangle}.$$



Step 2: Magnitude and unit direction In general, $|\mathbf{G}| = 12r/r^3 = 12/r^2$. Thus

$$|\mathbf{G}(P)| = \boxed{\frac{4}{3}}, \quad \widehat{\mathbf{G}(P)} = \boxed{\left\langle -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle}.$$

Step 3: Interpretation The negative sign reverses the outward position direction, so gravity points toward the origin. The denominator vanishes only at the origin; hence the domain is

$$\boxed{\mathbb{R}^3 \setminus \{(0, 0, 0)\}}.$$

Verification The displayed unit vector has length 1, and multiplying it by $4/3$ reproduces $\mathbf{G}(P)$.