

Question 3

For

$$\mathbf{F}(x, y) = \frac{\langle x, y \rangle}{x^2 + y^2},$$

analyze the field geometrically.

Tasks

1. State its domain and find its magnitude at a point of radius r .
2. Determine its direction and compare vectors at $(1, 0)$, $(2, 0)$, and $(0, -2)$.
3. Describe what happens as r approaches 0 and as r grows.

Question 3 – Solution

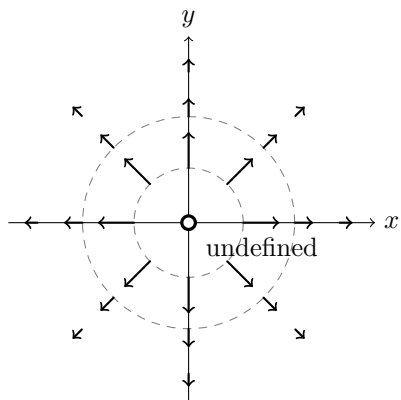
Strategy. Factor the position vector into its length and radial unit vector.

Step 1: Polar form With $r = \sqrt{x^2 + y^2}$ and $\mathbf{e}_r = \langle x/r, y/r \rangle$,

$$\mathbf{F} = \frac{r\mathbf{e}_r}{r^2} = \frac{1}{r}\mathbf{e}_r.$$

Hence the domain is $\mathbb{R}^2 \setminus \{(0, 0)\}$, the direction is radially outward, and

$$|\mathbf{F}| = \frac{1}{r}.$$



outward direction; magnitude $1/r$

Step 2: Samples

$$\mathbf{F}(1, 0) = \langle 1, 0 \rangle, \quad \mathbf{F}(2, 0) = \left\langle \frac{1}{2}, 0 \right\rangle, \quad \mathbf{F}(0, -2) = \left\langle 0, -\frac{1}{2} \right\rangle.$$

Doubling the radius halves the arrow length while preserving radial direction.

Step 3: Limiting behavior As $r \rightarrow 0^+$, $|\mathbf{F}| = 1/r \rightarrow \infty$, explaining the missing origin. As $r \rightarrow \infty$, the magnitude tends to 0.

Verification Multiplying any vector by its radius gives $\mathbf{e}_r$, a unit vector, so both the direction and inverse-radius magnitude are confirmed.