

Question 2

Determine the domain of the vector field

$$\mathbf{F}(x, y) = \left\langle \ln(x - y), \sqrt{4 - x^2 - y^2} \right\rangle.$$

Tasks

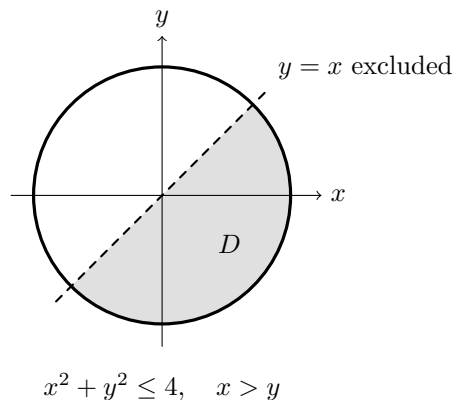
1. Express the domain as simultaneous inequalities.
2. Decide which parts of the line $y = x$ and circle $x^2 + y^2 = 4$ are included.
3. State where the field is continuous and where both components are differentiable.

Question 2 – Solution

Strategy. Intersect the natural domains of the logarithm and square root, keeping track of strict versus non-strict inequalities.

Step 1: Component restrictions The logarithm requires $x - y > 0$, while the square root requires $4 - x^2 - y^2 \geq 0$. Therefore

$$D = \{(x, y) : x > y, x^2 + y^2 \leq 4\}.$$



Step 2: Boundary decisions No point of $y = x$ is included because $\ln 0$ is undefined. The circular boundary is included only where $x > y$; there the second component equals 0 and the first remains defined.

Step 3: Regularity Both component functions are continuous wherever they are defined, so $\mathbf{F}$ is continuous on all of D . Both are differentiable where their defining inequalities are strict:

$$x > y, \quad x^2 + y^2 < 4.$$

At the included circular arc, the square-root component is continuous but its derivatives become singular.

Verification The shaded half-disk excludes its straight diagonal edge but includes the appropriate circular arc, reflecting $>$ versus $\leq$.