

**Question 1**

Consider the planar vector field

$$\mathbf{F}(x, y) = \langle x, -y \rangle.$$

**Tasks**

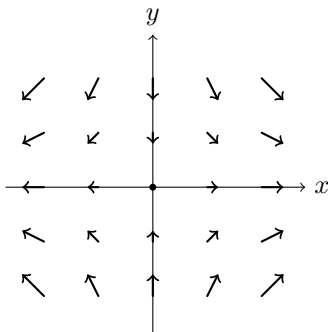
1. Evaluate  $\mathbf{F}$  at  $(1, 2)$ ,  $(-2, 1)$ , and  $(0, 0)$ .
2. Find the magnitude and direction of  $\mathbf{F}(1, 2)$ .
3. Describe the field on the coordinate axes and identify every zero vector.

## Question 1 – Solution

**Strategy.** Evaluate the two components pointwise, then use the component signs to understand the field globally.

**Step 1: Sample vectors**

$$\mathbf{F}(1, 2) = \langle 1, -2 \rangle, \quad \mathbf{F}(-2, 1) = \langle -2, -1 \rangle, \quad \mathbf{F}(0, 0) = \langle 0, 0 \rangle.$$



$$\mathbf{F}(x, y) = \langle x, -y \rangle$$

**Step 2: Magnitude and direction** At  $(1, 2)$ ,

$$|\mathbf{F}| = \sqrt{1^2 + (-2)^2} = \boxed{\sqrt{5}}, \quad \hat{\mathbf{F}} = \boxed{\left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right\rangle}.$$

**Step 3: Axis behavior and zeros** On the  $x$ -axis,  $\mathbf{F}(x, 0) = \langle x, 0 \rangle$ , so arrows point away from the origin. On the  $y$ -axis,  $\mathbf{F}(0, y) = \langle 0, -y \rangle$ , so arrows point toward the origin. Finally,

$$\mathbf{F}(x, y) = \mathbf{0} \iff x = 0 \text{ and } y = 0.$$

Thus the origin is the field's only zero.

**Verification** The plotted arrows reverse horizontally when  $x$  changes sign and vertically when  $y$  changes sign, exactly matching the formula.