

Question 9

Find the area of the saddle

$$z = \frac{x^2 - y^2}{2}$$

above the disk $x^2 + y^2 \leq 3$.

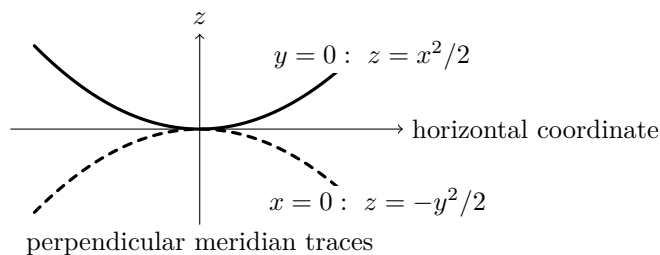
Tasks

1. Relate the two perpendicular meridian traces to the saddle shape.
2. Show that the surface-area factor is radial.
3. Evaluate the area exactly.

Question 9 – Solution

Strategy. Although the height changes sign by direction, the squared gradient depends only on distance from the origin.

Step 1: Geometry



Along $y = 0$ the trace opens upward, while along $x = 0$ it opens downward. Differentiation gives

$$f_x = x, \quad f_y = -y, \quad \sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + r^2}.$$

Step 2: Polar integral

$$\begin{aligned} S &= \int_0^{2\pi} \int_0^{\sqrt{3}} \sqrt{1 + r^2} r \, dr \, d\theta \\ &= 2\pi \left[\frac{(1 + r^2)^{3/2}}{3} \right]_0^{\sqrt{3}} \\ &= \frac{2\pi}{3} (8 - 1) = \boxed{\frac{14\pi}{3}}. \end{aligned}$$

Verification The projected disk has area 3π , while the area factor ranges from 1 to 2. Hence $3\pi < S < 6\pi$, and $14\pi/3$ lies in that interval.