

Question 8

For constants a, b, c , the plane

$$z = ax + by + c$$

lies above the unit disk $x^2 + y^2 \leq 1$. Its surface area there is 3π .

Tasks

1. Determine all possible gradient pairs (a, b) .
2. Explain the role of c .
3. Give two distinct planes satisfying the condition and verify them.

Question 8 – Solution

Strategy. A plane has a constant area-enlargement factor determined only by the magnitude of its gradient.

Step 1: Area equation Since $f_x = a$ and $f_y = b$,

$$S = \iint_D \sqrt{1 + a^2 + b^2} \, dA = \pi \sqrt{1 + a^2 + b^2}.$$

The condition $S = 3\pi$ gives

$$1 + a^2 + b^2 = 9, \quad \boxed{a^2 + b^2 = 8}.$$

Thus all possible gradients form the circle of radius $2\sqrt{2}$ in the ab -plane.

Step 2: Vertical translation The constant c does not appear in either derivative, so

$$\boxed{c \text{ is arbitrary}}.$$

Changing c translates the patch vertically without stretching it.

Step 3: Examples and verification Two choices are

$$\boxed{z = 2\sqrt{2}x} \quad \text{and} \quad \boxed{z = 2x + 2y + 5}.$$

Their gradient-square sums are respectively $8 + 0$ and $4 + 4$. Each therefore has area factor $\sqrt{1 + 8} = 3$ and area 3π above the unit disk.